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ANALYSIS OF THE PROCESSES OF HEAT TRANSFER WITH PERIODIC INTENSITY WITH ALLOWANCE FOR TEMPERATURE FLUCTUATIONS IN THE HEAT CARRIER

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A previously developed method of analysis of the processes of heat transfer with periodic intensity is generalized to the case of simultaneous fluctuations of the temperatures of the wall and the liquid. A final expression for the dependence of the coefficient of conjugation on the generalized parameter of the problem is obtained.

Processes of Heat Transfer with Periodic Intensity. For a large class of processes of convective heat transfer we can indicate the following two characteristic features: 1) periodic or random fluctuations of the parameters of the heat carrier (velocity, pressure, temperature, vapor content, phase interface); 2) mutual effect of the averaged and fluctuating temperature fields in the heat carrier and the wall (a conjugate convective-conductive character of the heat transfer). Account for them was a basis of the approximate theory of the processes of heat transfer with periodic intensity suggested in [1], where a boundary-value problem for the equation of heat conduction in a wall with a periodic boundary condition of the third kind on the heat-exchange surface was considered as a simplified scheme of a conjugate problem. The effect of the heat carrier on the wall is replaced by the "true" coefficient of heat transfer α:

$$\alpha = \frac{q_w}{\vartheta_w - \vartheta_\infty}, \tag{1}$$

its value averaged over a period is

$$\langle \alpha \rangle = \left\langle \frac{q_w}{\vartheta_w - \vartheta_\infty} \right\rangle. \tag{2}$$

In a traditional heat-transfer experiment and applied calculations, the average coefficient of heat transfer means

$$\alpha_m = \frac{\langle \theta_\omega \rangle}{\langle \vartheta_w - \vartheta_\infty \rangle}. \tag{3}$$

It is convenient to characterize the quantitative difference in the values of α averaged according to laws (2) and (3) by the "coefficient of conjugation"

$$\varepsilon = \frac{\alpha_m}{\langle \alpha \rangle}. \tag{4}$$

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Work [1] sought to determine the coefficient of conjugation ε for various thermophysical properties and thicknesses of the wall for ϑ_∞ = const. This case is physically consistent with flow of a heat carrier of a constant mass-mean temperature past a wall of finite thermal conductivity.

The present work is devoted to generalization of the approach presented in [1] to the case of simultaneous fluctuations of the temperatures of the wall and the liquid.

We represent all the quantities entering (1) as sums of stationary and fluctuating components:

$$\alpha = \langle\alpha\rangle + \alpha' ; \quad \vartheta_w = \langle\vartheta_w\rangle + \vartheta_w' ; \quad \vartheta_\infty = \langle\vartheta_\infty\rangle + \vartheta_\infty' ; \quad q_w = \langle q_w\rangle + q_w' .$$

Introducing dimensionless fluctuations of the quantities

$$\tilde{\alpha}' = \frac{\alpha'}{\langle\alpha\rangle} ; \quad \tilde{\vartheta}_w' = \frac{\vartheta_w'}{\langle\vartheta_w\rangle - \langle\vartheta_\infty\rangle} ; \quad \tilde{\vartheta}_\infty' = \frac{\vartheta_\infty'}{\langle\vartheta_w\rangle - \langle\vartheta_\infty\rangle} ; \quad \tilde{q}_w' = \frac{q_w'}{\langle q_w\rangle} ,$$

we rewrite the boundary condition of the third kind (1) in the form

$$(1 + \tilde{\alpha}')(1 + \tilde{\vartheta}_w' - \tilde{\vartheta}_\infty') = \varepsilon (1 + \tilde{q}_w') . \tag{5}$$

Averaging (5) over a period yields

$$\varepsilon = 1 + \langle \tilde{\alpha}' \tilde{\vartheta}_w' \rangle - \langle \tilde{\alpha}' \tilde{\vartheta}_\infty' \rangle . \tag{6}$$

According to (6), the effect of problem conjugation – the difference of the experimental value of the coefficient of heat transfer α_m from the true averaged value $\langle\alpha\rangle$ – is determined by the correlations of the fluctuations of the true coefficient of heat transfer $\tilde{\alpha}'$, the wall temperature $\tilde{\vartheta}_w'$, and the mass-mean temperature of the heat carrier $\tilde{\vartheta}_\infty'$. The limits of variation of ε are determined by the double inequality proved in [1] (in general form)

$$\left\langle \frac{1}{1 + \tilde{\alpha}'} \right\rangle^{-1} \le \varepsilon \le 1 . \tag{7}$$

It follows from (7) that the "experimental" coefficient of heat transfer is smaller than the "true" averaged one (equality of them is reached in the limiting case of an infinitely heat-conducting wall).

Constant Mass-Mean Temperature of the Heat Carrier. Assuming $\tilde{\vartheta}_\infty'$ = 0 in (5) and (6), we obtain the limiting case ϑ_∞ = const studied in [1]:

$$(1 + \tilde{\alpha}')(1 + \tilde{\vartheta}_w') = \varepsilon (1 + \tilde{q}_w') ; \tag{8}$$

$$\varepsilon = 1 + \langle \tilde{\alpha}' \tilde{\vartheta}_w' \rangle . \tag{9}$$

Here, as the intensity of the heat transfer increases ($\tilde{\alpha}' \ge 0$), the wall "is cooled" ($\tilde{\vartheta}_w' \le 0$), and as it decreases ($\tilde{\alpha} \le 0$), the wall, conversely, "is superheated" ($\tilde{\vartheta}_w' \ge 0$). As a consequence, we always have from (9)

$$\langle \tilde{\alpha} \tilde{\vartheta}_w \rangle \le 0 ; \quad \varepsilon \le 1 . \tag{10}$$

Constant Wall Temperature. Assuming $\tilde{\vartheta}_w'$ = 0 in (5) and (6), we obtain the limiting case ϑ_w = const:

$$(1 + \tilde{\alpha}')(1 - \tilde{\vartheta}_\infty') = \varepsilon (1 + \tilde{q}_w') ; \tag{11}$$

$$\varepsilon = 1 - \langle \tilde{\alpha}' \tilde{\vartheta}'_\infty \rangle , \tag{12}$$

which physically corresponds to evaporation of a thin film of liquid from the surface of an infinitely heat-conducting wall (provided that the mass flow rate of the vapor at both the inlet to and the outlet from the channel is kept constant). Here, an increase in the intensity of the heat transfer ($\tilde{\alpha}' \geq 0$) leads to an increase in the mass flow rate of transverse injection of the vapor into the flow core. This results in an increase in the density of the vapor and, consequently, in its pressure. The pressure increase, in turn, is accompanied by an increase in the temperature of vapor saturation ($\tilde{\vartheta}'_\infty \geq 0$). The temperature of the saturated vapor decreases ($\tilde{\vartheta}'_\infty \leq 0$) with the heat-transfer intensity ($\tilde{\alpha} \leq 0$) by virtue of the same reasons. Thus, we always have from (12)

$$\langle \tilde{\alpha}' \tilde{\vartheta}'_\infty \rangle \geq 0 ; \quad \varepsilon \leq 1 . \tag{13}$$

Exact Solutions for the Limiting Cases. We specify a stepwise law of fluctuations of the true coefficient of heat transfer

$$0 \leq \tau \leq \frac{\tau_0}{2} : \ \tilde{\alpha}' = (1 - b) ; \ \frac{\tau_0}{2} \leq \tau \leq \tau_0 : \ \tilde{\alpha}' = (1 + b) . \tag{14}$$

The minimum possible value of the coefficient of conjugation is determined by the left-hand side of inequality (7):

$$\varepsilon_{\min} = 1 - b^2 . \tag{15}$$

First, we consider the limiting case ϑ_w = const.

The variations in the vapor density ρ_v in the volume of the heat carrier are related to the fluctuations of the heat-flux density by

$$\frac{d\rho_v}{d\tau} = \frac{4q'_w}{rD} . \tag{16}$$

Expressing the pressure of the vapor (an ideal gas) from the equation of state (in view of the Truton rule) and using a linear approximation of the saturation curve, from (16) we obtain the following equation for the variation in the heat-carrier temperature:

$$\frac{d\vartheta'_\infty}{d\tau} = 0.4 \frac{\vartheta_\infty q'_w}{rD\rho_v} . \tag{17}$$

The solution of the system of equations (11), (12), (14), and (17) has the following form:

$$0 \leq \tau \leq \frac{\tau_0}{2} :$$

$$\theta = \frac{(1 + b) [1 - \exp(-2A)] - 2b [1 - \exp[-(1 + b) A]] \exp[(1 - b) t]}{(1 - b^2) [1 - \exp(-2A)]} ;$$

$$\frac{\tau_0}{2} \leq \tau \leq \tau_0 :$$

$$\theta = \frac{(1 - b) [1 - \exp(-2A)] + 2b (1 - \exp[-(1 - b) A]) \exp[-(1 + b) t]}{(1 - b^2) [1 - \exp(-2A)]} ; \tag{18}$$

$$\varepsilon = \frac{(1-b^2)^2 A \,\mathrm{th}\, A}{(1-b^2) A \,\mathrm{th}\, A - 2b^2\left(1 - \dfrac{\mathrm{ch}\, bA}{\mathrm{ch}\, A}\right)} . \tag{19}$$

Here $\theta = \langle\alpha\rangle(\vartheta_w - \vartheta_\infty)/\langle q_w\rangle$ is the dimensionless drop in temperature; $t = 0.8\langle\alpha\rangle\tau/(r\rho_v D)$ is the dimensionless time;

$$A = \frac{0.4\,\langle\alpha\rangle\,\tau_0}{r\rho_v D} \tag{20}$$

is the main parameter of the problem (the parameter of conjugation).

We now consider the limiting case ϑ_∞ = const in the approximation of a "thin wall": $\delta << \sqrt{a_w\tau_0}$. It is of interest to note that here we again obtain solution (18) and (19) for the temperature drop and ε, where the dimensionless time and the parameter of conjugation are written in the form

$$t = 2\,\langle\alpha\rangle\,\tau/(\rho_w c_w \delta)\,; \quad A = \frac{\langle\alpha\rangle\,\tau_0}{\rho_w c_w \delta} . \tag{21}$$

Approximate Solution for the General Case. In [1], the exact analytical solution for the coefficient of conjugation in the limiting case ϑ_∞ = const written in the form of the sum of an infinite functional series is obtained. Different versions of an approximate solution are suggested in [2-5]; in particular, it is shown in [2-4] that ε can be represented in the form of (19) with the parameter of conjugation

$$A = \frac{\langle\alpha\rangle\,\sqrt{a_w\tau_0}}{\lambda_w}\,\mathrm{cth}\,\frac{\delta}{\sqrt{a_w\tau_0}} . \tag{22}$$

When $\delta << \sqrt{a_w\tau_0}$, asymptotic form for an infinitely thin wall (21) follows from (22), and when $\delta >> \sqrt{a_w\tau_0}$ – the asymptotic form for a semiinfinite mass

$$A = \frac{\langle\alpha\rangle\,\sqrt{a_w\tau_0}}{\lambda_w} . \tag{23}$$

Application of the approximate method of [5] to the case under consideration allows the following generalized value of the parameter of conjugation in the exact solution (19) to be written:

$$A^2 \approx \left(\frac{0.4\,\langle\alpha\nabla\tau_0}{r\rho_v D}\right)^2 + \left(\frac{\langle\alpha\nabla\,\sqrt{a_w\tau_0}}{\lambda_w}\,\mathrm{cth}\,\frac{\delta}{\sqrt{a_w\tau_0}}\right)^2 . \tag{24}$$

Relations (19) and (24) allow one to calculate the coefficient of conjugation in the general case of the presence of temperature fluctuations in the wall and the heat carrier. As follows from (7) and (15), the value of ε varies within the limits

$$1 - b^2 \le \varepsilon \le 1 , \tag{25}$$

and therefore, it is convenient to represent its dependence on the parameter of conjugation in the "reduced form"

$$\tilde{\varepsilon} = \frac{\varepsilon - \varepsilon_{\min}}{1 - \varepsilon_{\min}} = \frac{2\,(1-b^2)\left(1 - \dfrac{\mathrm{ch}\, b\,A}{\mathrm{ch}\, A}\right)}{(1-b^2) A \,\mathrm{th}\, A - 2b^2\left(1 - \dfrac{\mathrm{ch}\, b\,A}{\mathrm{ch}\, A}\right)} . \tag{26}$$

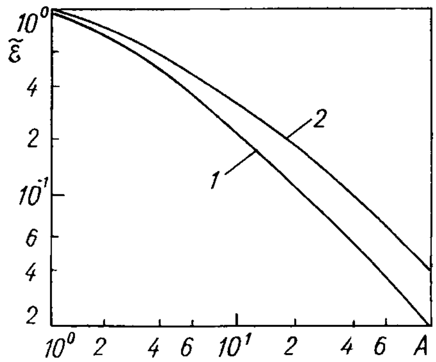

Fig. 1. Dependence of the reduced coefficient of conjugation on the generalized parameter: 1) relation (27) ($b \to 0$); 2) relation (28) ($b \to 1$).

When $b \to 0$ (the case of degeneration of the fluctuations of the true coefficient of heat transfer), it follows from (26) that

$$\tilde{\varepsilon} = \frac{2}{A} \operatorname{th} \frac{A}{2} . \tag{27}$$

When $b \to 1$ (the case of the maximum possible fluctuations of the true coefficient of heat transfer), we obtain

$$\tilde{\varepsilon} = \frac{4}{3 + A \operatorname{cth} A} . \tag{28}$$

Figure 1 shows the limiting dependences $\tilde{\varepsilon}(A)$ according to (27) and (28). The entire dependences $\tilde{\varepsilon}(A)$ for region (25) lie within the indicated limits.

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NOTATION

τ, time; τ_0, period of the fluctuations; ϑ, temperature; q, heat-flux density; α, true coefficient of heat transfer; $\langle\alpha\rangle$, true averaged coefficient of heat transfer; α_m, experimental coefficient of heat transfer; ε, coefficient of conjugation; b, dimensionless amplitude of the fluctuations of the true coefficient of heat transfer; ρ_v, vapor density; ρ_w, c_w, λ_w, and a_w, density, specific heat capacity, thermal conductivity, and thermal diffusivity of the wall, respectively; r, specific heat of the phase transition; D, hydraulic diameter of the channel; δ, wall thickness. Subscripts: w, conditions on the wall; ∞, conditions in the heat carrier ("at infinity"); $\langle\ \rangle$, averaging over a period of the fluctuations; prime, fluctuating quantity; tilde, dimensionless quantity.

REFERENCES

1. D. A. Labuntsov and Yu. B. Zudin, *Processes of Heat Transfer with Periodic Intensity* [in Russian], Moscow (1984).
2. Yu. B. Zudin, *Teplofiz. Vys. Temp.*, **29**, No. 5, 935-940 (1991).
3. Yu. B. Zudin, *Dokl. Ross. Akad. Nauk*, **360**, No. 3, 344-345 (1998).
4. Yu. B. Zudin, *Inzh.-Fiz. Zh.*, **68**, No. 2, 225-228 (1995).
5. Yu. B. Zudin, in: *Heat Conduction and Thermal Insulation, Proc. 2nd Russian Nat. Conf. on Heat Transfer* [in Russian], Moscow (1998), Vol. 7, pp. 99-102.